Air Traffic, Boarding and Scaling Exponents

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Air traffic – Vehicular traffic

- The growing need for mobility through the world shows no sign of slowing down.
- The air traffic is a very important part of the global transportation network.
- In distinction from vehicular traffic, the boarding of an airplane is a significant part of the whole transportation process.
- Here we study an airplane boarding model, introduced earlier by Frette and Hemmer (2012), with the aim to determine precisely the asymptotic power–law scaling behavior of the mean boarding time $\langle t_b \rangle$ and other related quantities for large number of passengers N.

Time needed to board an airplane

Recently, an airplane boarding model has been proposed by Hemmer and Frette, Phys. Rev. E **85**, 011130 (2012).

- N passengers have reserved seats, but enter the airplane in arbitrary order (N! permutations).
- This model shows a power–law scaling behaviour $\langle t_b \rangle \propto N^\alpha$ of the mean boarding time $\langle t_b \rangle$ depending on N. The exponent $\alpha = 0.69 \pm 0.01$ was found by Hemmer and Frette, considering system sizes $2 \leq N \leq 16$.
- However, later it was found out that the effective scaling behaviour changes with N, and $\alpha = 1/2$ is the true exponent, describing the power law at $N \to \infty$.
- We have explained it by corrections to scaling.

Scaling exponents

Power–law scaling is analysed, taking into account corrections to scaling, as in critical phenomena. Our numerical results are perfectly consistent with

$$\langle t_b\rangle(N) \propto N^{\alpha}\left(1 + bN^{-\theta} + cN^{-2\theta} + o\left(N^{-2\theta}\right)\right), \qquad (1)$$

$$\langle\overline{\Delta n}\rangle(N) \propto N^{\nu}\left(1 + dN^{-\theta} + eN^{-2\theta} + o\left(N^{-2\theta}\right)\right), \qquad (2)$$

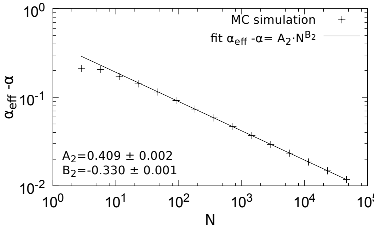
where $\langle\overline{\Delta n}\rangle(N)$ is the mean number of passengers taking seats simultaneously. It corresponds to

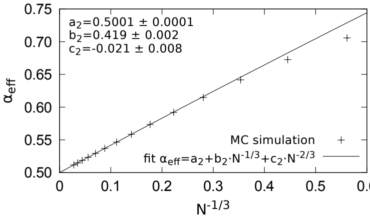
$$\alpha_{\mathrm{eff}}(N) = \alpha + b'N^{-\theta} + c'N^{-2\theta} + o\left(N^{-2\theta}\right) , \qquad (3)$$

$$\nu_{\mathrm{eff}}(N) = \nu + d'N^{-\theta} + e'N^{-2\theta} + o\left(N^{-2\theta}\right) \qquad (4)$$

We find $\alpha = \nu = 1/2$: a classical value, and $\theta \approx 1/3$.

MC estimation of the exponents

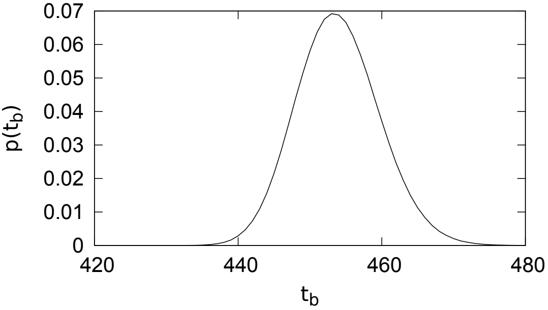




Determination of the exponent α and the correction–to–scaling exponent θ. The left figure is used to determine the correction–to–scaling exponent θ. From linear fit in log–log plot we see that $\theta \approx 1/3$. The right figure is used to determine the exponent α. From quadratic fit we see that $\alpha = 0.5001 \pm 0.0001 \approx 1/2$. Simulations for $N \leq 2^{16} = 65\,536$ have been performed by Martins Brics. The material is partly published in Phys. Rev. E **87**, 042117 (2013) by Brics at al.

Probability distribution of the boarding time

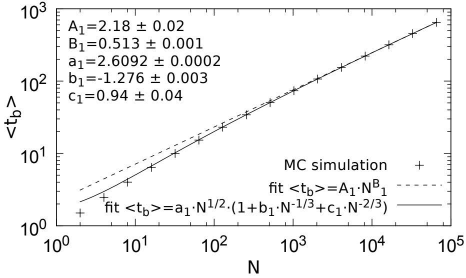
For $N = 2^7 = 128$ passengers $\langle t_b \rangle$ is about 7 min, if one time step is 20 s.
For $N = 2^{15} = 128 \cdot 256$ $\langle t_b \rangle$ is not 7 min $\cdot$ 256 $\approx$ 30 h, but ...



Probability distribution $p(t_b)$ over boarding time steps t_b determined by MC simulations in the case of $N = 2^{15} = 32\,768$ passengers. It shows $\langle t_b \rangle \approx 20\text{ s} \cdot 455 \approx 2.5\text{ h}$.

Conclusions

An airplane boarding model, showing a power–law scaling behaviour,



has been considered as a toy example, clearly demonstrating the importance of corrections to scaling and the necessity to consider very large systems to obtain correct values of the exponents.